Energy diffusion in strongly driven quantum chaotic systems: the role of correlations of the matrix elements

2008 J. Phys. A: Math. Theor. 41 055103

(http://iopscience.iop.org/1751-8121/41/5/055103)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **41** (2008) 055103 (13pp) doi:10.1088/1751-8113/41/5/055103

Energy diffusion in strongly driven quantum chaotic systems: the role of correlations of the matrix elements

P V Elyutin and A N Rubtsov

Department of Physics, Moscow State University, Moscow 119992, Russia

E-mail: pve@shg.phys.msu.su

Received 9 November 2007, in final form 14 December 2007
Published 23 January 2008
Online at stacks.iop.org/JPhysA/41/055103

Abstract

The energy evolution of a quantum chaotic system under the perturbation that harmonically depends on time is studied for the case of large perturbation, in which the rate of transition calculated from the Fermi golden rule (FGR) is about or exceeds the frequency of perturbation. For this case, the models of the Hamiltonian with random non-correlated matrix elements demonstrate that the energy evolution retains its diffusive character, but the rate of diffusion increases slower than the square of the magnitude of perturbation, thus destroying the quantum–classical correspondence for the energy diffusion and the energy absorption in the classical limit $\hbar \to 0$. The numerical calculation carried out for a model built from the first principles (the quantum analog of the Pullen–Edmonds oscillator) demonstrates that the evolving energy distribution, apart from the diffusive component, contains a ballistic one with the energy dispersion that is proportional to the square of time. This component originates from the chains of matrix elements with correlated signs and vanishes if the signs of matrix elements are randomized. The presence of the ballistic component formally extends the applicability of the FGR to the non-perturbative domain and restores the quantum–classical correspondence.

PACS number: 05.45.−a

1. Introduction

The problem of susceptibility of chaotic systems to perturbations has attracted much attention in the last decade [1–9]. This problem is fundamental, since it includes the determination of the response of a material system to an imposed external electromagnetic field, the setup that is typical for many experiments. Due to the sensitivity of classical phase trajectories or quantum energy spectra and stationary wavefunctions of chaotic systems to small changes of their parameters, the problem is challengingly difficult. A consistent and noncontroversial

picture covering (albeit qualitatively) all the essential cases of the response has not been yet drawn at present. From the point of view of general theory, the problem is related to the applicability of the concept of quantum–classical correspondence to chaotic systems, that is a long-standing question in its own right [10, 11].

We shall study a one-particle system with the Hamiltonian of the form $\hat{H} = \hat{H}_0 - F\hat{x}\cos\omega t$, where $\hat{H}_0(\hat{\mathbf{p}}, \hat{\mathbf{r}})$ is the Hamiltonian of the unperturbed system; $\hat{\mathbf{p}}$ and $\hat{\mathbf{r}}$ are the operators of the Cartesian components of the momentum and of the position of the particle. The classical system with the Hamiltonian function $H_0(\mathbf{p}, \mathbf{r})$ will be assumed strongly chaotic, that is, nearly ergodic on the energy surfaces in a wide range of energy values. In the perturbation operator $\hat{V}(t) = -F\hat{x}\cos\omega t$, the active variable $\hat{x}$ is one of the Cartesian coordinates of the particle, coupled to the external homogeneous force field. The amplitude F in the following will be referred to as the field. We shall deal with the quasiclassical case, when the Planck constant $\hbar$ is small in comparison with the action scale of the system H_0.

Under the influence of the perturbation, the energy value $E(t) = H_0(t)$ varies in a quasirandom way. These variations can be frequently described as a process of the energy diffusion [12, 13], when for the ensemble with the microcanonical initial energy distribution $H_0(0) = E$ the dispersion of the energy increases linearly with time, $\langle \Delta E^2(t) \rangle = 2Dt$, where $D(E, F, \omega)$ is the energy diffusion coefficient.

If the external field F is sufficiently small in comparison with the appropriately averaged values of the forces acting on a particle in the unperturbed system, then in the classical model the energy diffusion coefficient D can be expressed through the characteristics of the unperturbed chaotic motion of the active coordinate [9]. If $x(E, t)$ is the law of the chaotic motion on the energy surface $H = E$, then its autocorrelation function is defined by the equation

$$B_x(E, \tau) = \overline{x(E, t)x(E, t + \tau)} - [\overline{x(E, t)}]^2, \tag{1}$$

where the overline denotes the time averaging. The power spectrum of the coordinate $S_x(E, \omega)$ for the motion on the surface with the constant energy value E is the Fourier transform of the autocorrelation function

$$S_x(E, \omega) = \frac{1}{2\pi} \int B_x(E, \tau)\, \mathrm{e}^{-\mathrm{i}\omega\tau}\, \mathrm{d}\tau. \tag{2}$$

The energy diffusion coefficient is given by the expression

$$D = \frac{\pi}{2}\omega^2 F^2 S_x(E, \omega). \tag{3}$$

The same expression (3) can be obtained in the case of weak perturbation in the classical limit from the quantum model. The evolution of the quantum system can be treated as a sequence of one-photon transitions between stationary states of the unperturbed system $|n\rangle \to |k\rangle$, accompanied with the absorption or emission of the quanta $\hbar\omega$. For small $\hbar$, the energy spectrum of $\hat{H}_0$ becomes very dense, thence the rates of transition are given by the Fermi golden rule (FGR)

$$\dot{W}_F = \frac{\pi}{2\hbar} F^2 |x_{nk}|^2 \rho(E_k), \tag{4}$$

where x_{nk} is the matrix element of the active coordinate, and $\rho(E_k)$ is the density of states near the final state of the transition. The condition of the formation of the quasicontinuum, $F|x_{nk}|\rho(E_k) \gg 1$, that permits the use of equation (4), is always fulfilled in the classical limit, since its lhs scales as $\hbar^{-\frac{d+1}{2}}$, where d is the number of degrees of freedom.

Although the matrix elements x_{nk} in quantum chaotic systems fluctuate wildly with the variation of k [10, 11], the averaged-squared quantity $\overline{|x_{nk}|^2}$ in the limit $\hbar \to 0$ is smooth; it is proportional to the ratio of the power spectrum to the density of states,

$$\overline{|x_{nk}|^2} \approx \frac{S_x(E,\omega)}{\hbar\rho(E)}, \tag{5}$$

where $\omega = (E_k - E_n)/\hbar$ [14, 15]. From equations (4) and (5), we have for the transition rate

$$\dot{W}_F = \frac{\pi}{2\hbar^2} F^2 S_x(E,\omega). \tag{6}$$

Then for the energy dispersion for small t we have $\langle \Delta E^2 \rangle = 2D_F t = 2(\hbar\omega)^2 \dot{W}_F t$, that returns us to (3) for the energy diffusion coefficient. It can be shown that the same expression for D holds also for large t [9]. The energy absorption in chaotic systems comes as a consequence of the energy diffusion [4]. With the account of the dependence on the energy of the power spectrum $S_x(E,\omega)$ and the density of states $\rho(E)$ the diffusion becomes biased, and the energy absorption rate Q is given by the formula [2, 4]

$$Q = \frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}E}(\rho D). \tag{7}$$

Although for weak fields D does not depend on the Planck constant $\hbar$, the condition of the applicability of (4) does. The FGR is, after all, only a formula of the first-order perturbation theory. It is based on the assumption that the transition process has a resonant character—that the width Δ of the energy distribution of states populated from the original one, given by the Weisskopf–Wigner formula $\Delta = \hbar\dot{W}$ [16], is small in comparison with the quanta energy $\hbar\omega$. From (6) it is evident that in the classical limit $\hbar \to 0$ this condition will be violated. In the following, we shall use the border value of the field F_b, defined by the condition $\dot{W}_F(F_b) = \omega$, and refer to the domain $F \geqslant F_b$ as the range of the strong field.

By analogy with other models, for strong fields one can expect a slow-down of the growth of the energy diffusion coefficient D and of the energy absorption rate Q. For example, for a two-level system with relaxation the quadratic dependence on the absorption rate $Q \propto F^2$ for small field turns into a field-independent value Q_0 for the strong one. The border is determined by the condition $\Omega^2/\Gamma_1\Gamma_2 \sim 1$, where Ω is the Rabi frequency and Γ_1, Γ_2 are the longitudinal and transversal relaxation rates, correspondingly [17]. The rate of transitions from the discrete to continuous energy spectrum (that are basically covariant with the energy absorption rate Q), studied in the context of the theory of photoionization, for sufficiently strong fields can even decrease with the increase of F—the effect that is known as atom stabilization by the strong field [18].

The slow-down of the energy diffusion in strong harmonic fields for the model of quantum chaotic systems with random uncorrelated matrix elements has been first demonstrated by Cohen and Kottos [5]. A different approach [19] has lead to qualitatively the same results. This slow-down destroys the quantum–classical correspondence. As well the quantum corrections to the energy absorption rate in the model with the random-matrix Hamiltonian have been studied theoretically [20].

It has been demonstrated by Kottos and Cohen [21] that for the first-principles model constructed by the quantization of the Hamiltonian of a classically chaotic system, the response to a sudden change of the (otherwise) stationary Hamiltonian measured by the energy spreading restores its classical behaviour for sufficiently small values of $\hbar$ in contrast with the model with random independent matrix elements. We also note that some evidence of the extended domain of validity of the FGR has been obtained from the experimental studies of the dc conductivity in solids [22].

The purpose of the present paper is to study the restoration of the quantum–classical correspondence for the harmonically driven system.

2. Numerical experiment

The system chosen is the Pullen–Edmonds oscillator [23] that describes the two-dimensional motion of a particle in the quartic potential. The Hamiltonian of this system is

$$H_0 = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{m\omega_0^2}{2}\left(x^2 + y^2 + \frac{x^2y^2}{\lambda^2}\right). \tag{8}$$

In the following we use the particle mass m, the frequency of small oscillations ω_0 and the nonlinearity length λ as unit scales, and write all equations in the dimensionless form.

The properties of chaotic motion of the Pullen–Edmonds model are thoroughly studied [24–26]. With the increase of energy, the system becomes more chaotic both in extensive (that is characterized by the measure of the chaotic component $\mu_s(E)$ on the surface of the Poincaré section) and in intensive (that is measured by the magnitude of the Lyapunov exponent $\sigma(E)$) aspects. For values of energy $E > 2.1$ the measure $\mu_s > 0.5$, and chaos dominates in the phase space; for $E > 5$ the chaotic motion of the system is approximately ergodic [24].

The matrix of the quantum Hamiltonian operator of the model (6) has been calculated on the basis of the unperturbed two-dimensional isotropic harmonic oscillator for the value $\hbar = 0.05$. Due to the symmetry of the system, the submatrices with different parities of the quantum numbers n_x and n_y can be diagonalized separately.

By expanding the wavefunction of the system $\Psi(\mathbf{r}, t)$ on the basis of the eigenstates $\{\varphi_m\}$ of the system $\hat{H}_0$

$$\Psi(\mathbf{r}, t) = \sum_m a_m(t)\varphi_m(\mathbf{r})\,\mathrm{e}^{-\mathrm{i}\omega_m t}, \tag{9}$$

we obtain for the amplitudes a_m the system of equations

$$\mathrm{i}\frac{\mathrm{d}a_k}{\mathrm{d}t} = \sum_k \Omega_{km}\cos\omega t\,\mathrm{e}^{\mathrm{i}\omega_{km}t}a_m, \tag{10}$$

where the quantities $\Omega_{kn} = \hbar^{-1}Fx_{kn}$ are the Rabi frequencies of transitions. This system has been solved numerically for the 2352 amplitudes of the eigenstates with 'even–even' and 'odd–even' parities of n_x and n_y that include all states of these classes with energies in the band $10 \leqslant E \leqslant 12$.

For the initial conditions in the runs with different values of F and ω, we have used the same normalized narrow wavepacket with randomly chosen real amplitudes $a_k(0)$ with the complexity (inverse participation ratio) $C = \left(\sum_k a_k^4(0)\right)^{-1} = 21$, the mean initial energy $\langle E\rangle = E_0 = 11.0$ and the initial energy dispersion $\Delta E^2(0) = 2.5 \times 10^{-3}$.

To expose the role of correlations of the matrix elements x_{kn} on the energy kinetics, we compare the properties of the 'natural' system with its 'randomized' analog with the matrix elements $y_{mn} = x_{mn}A_{mn}$, where A_{mn} are the elements of a symmetric matrix that take values 1 or -1 at random with equal probabilities. This randomization, that has been introduced in [21], destroys the correlation of the matrix elements. The tests has shown that the results do not depend on the specific choice of the matrix A_{mn} within the limits of an error of about 1%.

Figure 1 presents the typical dependence on time of the energy dispersion for the natural (filled circles) and randomized (open circles) models. To extract the value of the energy diffusion coefficient, the numerical dependence was fitted by the law of evolution of the energy dispersion for the diffusion equation with the constant D on the interval $(-L, L)$ with

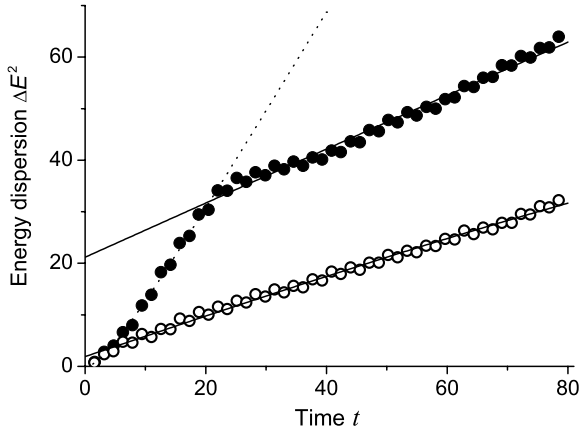

Figure 1. The dependence of the energy dispersion ΔE^2 on time t. The energy dispersion is measured in the units of $(\hbar\omega_0)^2$. The field strength corresponds to the Fermi point: $F = F_b = 0.022$; the perturbation frequency $\omega = 1.00$. Symbols present the numerically calculated values for the natural (filled circles) and randomized (open circles) models. Solid lines show the best fits with the numerical data by formula (11). Dashed line shows the best fit with the numerical data by formula (12).

impenetrable walls on the borders and the initial condition in the form of the $\delta(0)$ peak. This dependence can be described (with the local accuracy better than 3%) by the formula

$$\Phi(a, b; t) = a(1 - \exp(-bt))(1 + 0.633bt \exp(-1.161bt)), \tag{11}$$

where $a = L^2/3$ and $b = 6D/L^2$. The best fits of formula (11) with the numerical data are shown in figure 1 by solid lines. The initial moment t_0 has been used as the third fitting parameter.

For once, it is clearly seen that the randomization suppresses the process of the energy diffusion. Second, for the natural model one can see the presence of two different regimes—a fast initial diffusion sharply slows down at a crossover time $t_c \simeq 20$. Before the crossover time, the dependence of $\Delta E^2(t)$ for the natural models agrees with the predictions of the classical theory (see figure 2) with the average relative error about 9%, that is a reasonable accuracy of the agreement of the quantum model with $\hbar = 0.05$ and the classical limit.

We note that at the moment of crossover the energy dispersion $\Delta E^2(t_c) = 32(\hbar\omega_0)^2$ is much less than the saturated value $\Delta E_s^2 = a = 133(\hbar\omega_0)^2$, that corresponds to the uniform probability distribution throughout the band of the states taken into account. To understand what happens at the crossover time, we have to study more closely the time development of the probability distribution.

The overall form of the energy distribution for the natural model is rather accurately approximated by the Gaussian form that follows from the model of the energy diffusion with the constant D. This agreement can be seen in figure 3, where the logarithm of the energy density distribution is shown against the reduced energy shift $\Delta\varepsilon = (E - E_0)/\hbar\omega_0$, where E_0 is the mean energy value of the initial wavepacket.

Although the agreement seems to be very good, one must keep in mind that the vertical scale of the graph is logarithmic. By subtraction of the parabolic fit from the numerical distribution, we come to the picture of deviations that is shown in figure 4.

The peak at $\Delta\varepsilon = 0$ corresponds to the part of the initial packet that is not depleted yet by the energy spreading. Two bumps are clearly seen in the picture: their maxima are

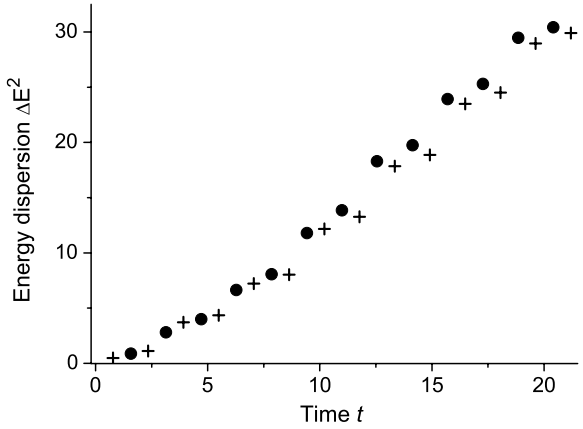

Figure 2. The dependence of the energy dispersion ΔE^2 on time t. The units and parameter values are the same as in figure 1. Symbols show the values calculated for the quantum natural (filled circles) and classical (crosses) models.

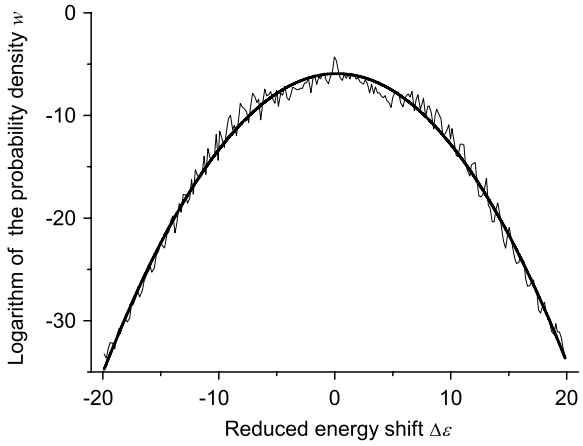

Figure 3. The logarithm of the energy density distribution for the time $t = 17.3$ for the parameters $F = 0.606F_b$, $\omega = 1.00$. The grassy line is the numerical data, the solid line presents the best fit of data with the parabola $p(\Delta\varepsilon) = \alpha - \beta(\Delta\varepsilon)^2$.

located at $\Delta\varepsilon \simeq \pm 12$. The calculations show that these bumps propagate outwards with a constant velocity, that is ballistically. This velocity linearly depends on the field strength F. The crossover time corresponds to the moment when these bumps reach the borders of the treated band of states. Therefore, only the part of the graph that precedes the t_c corresponds to the properties of the model; the second part is just an artifact of the truncation of the basis. To estimate the diffusion coefficient, we fitted the dependence of the initial state by the two-parameter formula

$$\phi(D, \tau; t) = \frac{2Dt^2}{\tau + t}, \tag{12}$$

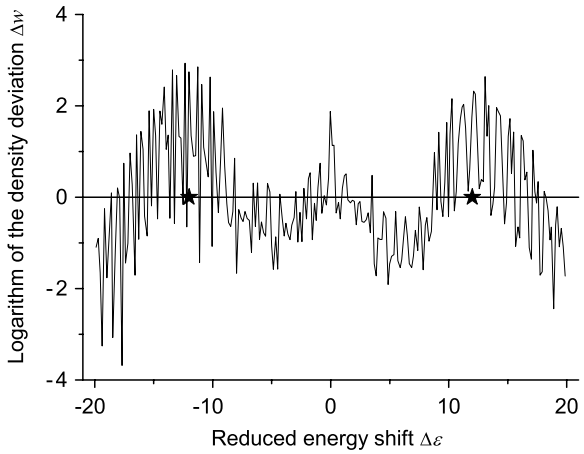

Figure 4. The deviation of the logarithm of the energy density distribution from the best-fitted parabola Δw as a function of the dimensionless energy deviation $\Delta\varepsilon$ for the same parameters as in figure 2. The black stars note the maxima of the ballistic bumps (see the text).

where the time shift τ accounts for the duration of the initial stage, when the law of the dispersion growth is always quadratic. The best-fitted function ϕ is plotted in figure 1 by the dashed line.

The ballistic spreading of the energy distribution in the time domain $t \gg 2\pi/\omega$ is well known for the model of a one-dimensional resonantly excited harmonic oscillator with the Hamiltonian $\hat{H}(t) = (\hat{p}^2 + \hat{x}^2)/2 - F\hat{x}\cos t$. In the quasiclassical domain (for large quantum numbers $n \gg 1$), the matrix elements of the coordinate can be taken constant, $x_{nm} = X(\delta_{n,n-1} + \delta_{n,n+1})$, where δ_{ij} is the Kronecker delta symbol. With this assumption in the rotating wave approximation for the initial condition $a_k(0) = \delta_{kn}$, the probabilities w_k to find the system in the state $|k\rangle$ are given by the well-known formula

$$w_k = |a_k(t)|^2 = J_{n-k}^2(\Omega t), \tag{13}$$

where $J_n(z)$ are the Bessel functions of the first kind and $\Omega = FX/2\hbar$ is the value of the Rabi frequency. Equation (13) yields for the energy dispersion the quadratic law,

$$\Delta E^2(t) = \tfrac{1}{2}(\hbar\omega_0\,\Omega t)^2. \tag{14}$$

However, the randomization of signs of the matrix elements does not influence the energy kinetics in this model.

The influence of randomization can be explained by the toy 'double ladder' model. This system has the doubly degenerate equidistant energy spectrum $E_n = \hbar\omega_0[n/2]$, where [,] denotes the integer part of the number. The matrix elements of the coordinate connect each state $|n\rangle$ to all four states $|m\rangle$ with the energy differences $E_n - E_m = \pm\hbar\omega_0$:

$$x_{nm} = X(\delta_{n,n-2} + \delta_{n,n-1} + \delta_{n,n+2} + \delta_{n,n+3}) \tag{15}$$

for even n, and

$$x_{nm} = X(\delta_{n,n-3} + \delta_{n,n-2} + \delta_{n,n+1} + \delta_{n,n+2}) \tag{16}$$

for odd n. In this model, for the resonant perturbation $\hat{V}(t) = -F\hat{x}\cos\omega_0 t$ the energy spreading is ballistic, $\Delta E^2(t) \propto t^2$, whereas the randomization of signs leads to the localization of the quasienergy states, and the energy dispersion growth saturates.

One can suppose that the ballistic component in the quantum chaotic model is carried through the subset of states that are similar to the 'double ladder' model. The degree of correlation of the matrix elements can be estimated from the construction

$$T_n = \sum_{m,l,k} x_{nm} x_{ml} x_{lk} x_{kn}, \tag{17}$$

that describes the sum of contributions of all possible four consequent transitions that start and end on the same state $|n\rangle$. The correlation index ν can be defined as the ratio of the average value of T for the randomized system to that of the natural system. For the 'double ladder' model, we have

$$\nu = \frac{\langle T_n^R \rangle}{\langle T_n^N \rangle} = \frac{7}{12} = 0.583. \tag{18}$$

The relatively large value of this number is explained by the large contribution of symmetric contours with $m = k$, that are invariant under the randomization.

For the Pullen–Edmonds model the value of the correlation index, calculated for the group of 1176 states in the middle of the band with different randomizing matrices A_{mn}, is $\nu = 0.760 \pm 0.004$. This value is rather close to that of the 'double ladder' model; that makes the analogy plausible.

By simple algebra one can show that $T_n = \langle n|x^4|n\rangle$. For the natural model one can calculate the classical limit of this quantity: since in this limit almost all wavefunctions become ergodic on the energy surface [27], we have $T^N(E) = \overline{x^4(E,t)}$. In the randomized case, after averaging equation (17) over all possible ways of randomization, only the terms with $m = k$ or $n = l$ will contribute to the sum:

$$T_n^R = \sum_{m,l} |x_{nm}|^2 |x_{ml}|^2 + \sum_{m,k} |x_{nm}|^2 |x_{nk}|^2. \tag{19}$$

In the classical limit for the ergodic wavefunctions, we have $T^R(E) = 2(\overline{x^2(E,t)})^2$. For the energy value $E = 11$, the direct calculations give $T^N = 31.24$, $T^R = 22.52$ and $\nu_c = 0.721$; thus, our numerical value of ν differs from its classical limit only by 5%.

It must be stressed that the combined effect of the bulk diffusion spreading and the overlaying ballistic bumps propagation produces the *linear* growth of the energy dispersion (see figure 1) that will be referred to as the effective diffusion. This phenomenon can be explained with a simple model. Let us assume that the distribution of the probability can be treated as a superposition of the diffusive part with the diffusion coefficient D_R that is equal to that of the randomized model, and of the two completely localized bumps that propagate with the constant velocity; their contribution to the probability density is $w_b = q\delta(\varepsilon + Vt) + q\delta(\varepsilon - Vt)$. The dependence of the energy dispersion on time for this model is $\Delta E^2(t) = 2D_R t + 2qV^2t^2$. We can estimate the bump fractions of probability q at crossover times from the numerical data. They have small values in the range from 0.003 to 0.04 for $\omega = 1.00$ and from 0.01 to 0.09 for $\omega = 1.62$. The time dependence of $\Delta E^2(t)$ is compatible with the observed linear growth if the bump fraction of the total probability density decreases with time by the hyperbolic law, $q \approx Q/t$. This decrease is due to the irreversible depletion of the probability amplitudes from 'the double ladder' channel to the array of the background states. The formation of the bumps have to take some time not less than the field period $T = 2\pi/\omega$. The hyperbolic extrapolation gives the values $q(T) = 0.10 \pm 0.05$ for $\omega = 1.00$ and $q(T) = 0.15 \pm 0.02$ for $\omega = 1.62$. Although the true picture of evolution of the probability distribution is considerably more complicated, these simple estimates show that relatively small bumps can produce an important, sometimes even dominating, contribution to the energy dispersion.

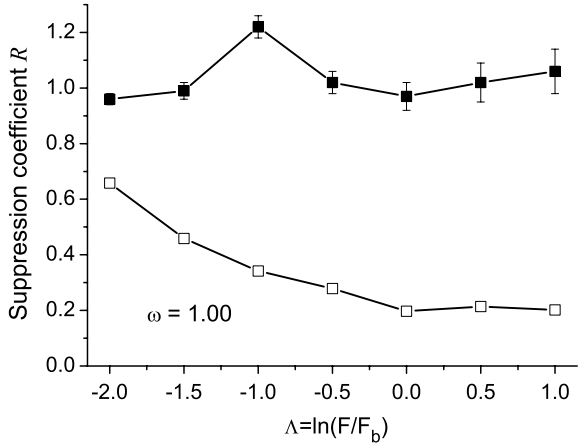

Figure 5. The dependence of the suppression coefficient R on the logarithm of the field strength $\Lambda = \ln(F/F_b)$ for the frequency of perturbation $\omega = 1.00$.

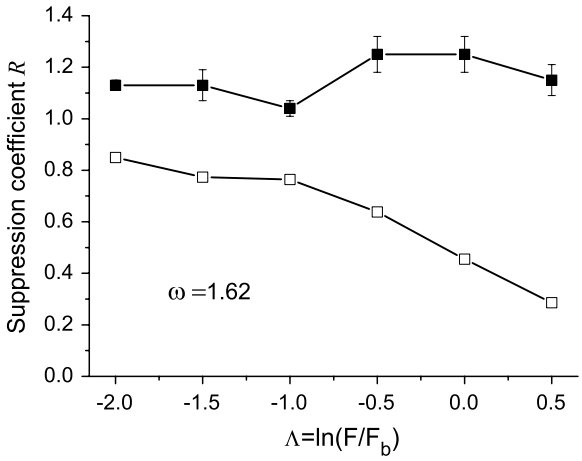

Figure 6. The dependence of the suppression coefficient R on the logarithm of the field strength $\Lambda = \ln(F/F_b)$ for the frequency of perturbation $\omega = 1.62$.

We define the suppression coefficient $R(F, \omega)$ as the ratio of the effective energy diffusion coefficient to its value D_F that follows from the FGR:

$$R(F, \omega) = \frac{D(F, \omega)}{2(\hbar\omega)^2 \dot{W}_F}. \tag{20}$$

The dependence of R on the field strength is shown in figures 5 and 6 for two different values of the perturbation frequency.

For the natural model R remains approximately constant with the value close to unity, whereas for the randomized model the energy diffusion slows down in qualitative agreement with the conclusions of [5, 19]. The difference between the behaviour of the natural and randomized models is connected with the possibility of the (constructive) interference of the amplitudes propagating via matrix elements with correlated signs. In the natural case, this

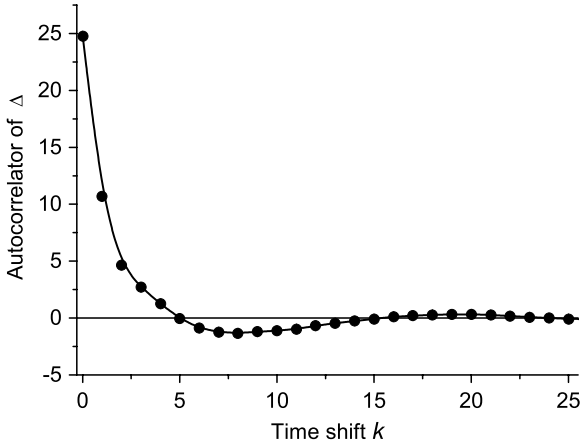

Figure 7. The dependence of the autocorrelator of the reduced energy increments $B_\Delta(k) = \langle \Delta_n \Delta_{n+k} \rangle$ on the time shift k measured in periods of the perturbation on the energy surface $E = 11.0$ for the perturbation frequency $\omega = 1.00$.

interference creates the ballistic bumps in the probability density, extends the applicability of the Fermi golden rule and provides the quantum–classical correspondence. The destruction of this interference by the randomization suppresses the rate of the energy diffusion in the strong fields.

We have observed no restrictions on the applicability of the FGR at large times up to the limit of our computational ability. Such restrictions could indicate the presence of the dynamical localization. It can be seen from figure 1 that after the crossover time the energy diffusion continues. The saturated values of $\Delta E^2(t = \infty)$ extrapolated from these parts of numerical data are close to the saturated value $\Delta E_s^2 = 133(\hbar\omega_0)^2$ that corresponds to the uniform probability distribution throughout the band of the states taken into account. Thus we have no indications of the dynamic localization whatsoever: the quasienergy states in the studied domain are either delocalized or localized in the range that is much wider than the treated band of states.

3. Energy diffusion in the classical model

The classical expression for the diffusion coefficient (3) is derived in the limit of the infinitesimal perturbation, when one can neglect the influence of the perturbation on the law of motion of the active coordinate $x(t)$. Let us study the formation of this coefficient. We represent the external field in the form $F \sin \omega t$ and denote by $t_n = 2\pi n/\omega$ the moments of time at which the external field take zero values. The variation of the energy for one field period between these moments is exactly proportional to the field strength,

$$\Delta E_n = F \int_{t_n}^{t_{n+1}} \dot{x}(t) \sin \omega t \, \mathrm{d}t = F \Delta_n. \tag{21}$$

The quantities Δ_n we shall call the reduced variations of the energy. In the accepted approximation, they do not depend on the field strength.

The following calculations were carried out for the Pullen–Edmonds oscillator on the energy surface $E = 11.0$ and for the perturbation frequency $\omega = 1.00$.

The values of the reduced energy increments Δ_n on the neighbouring time intervals are correlated. Figure 7 presents the form of the autocorrelation function of the reduced energy

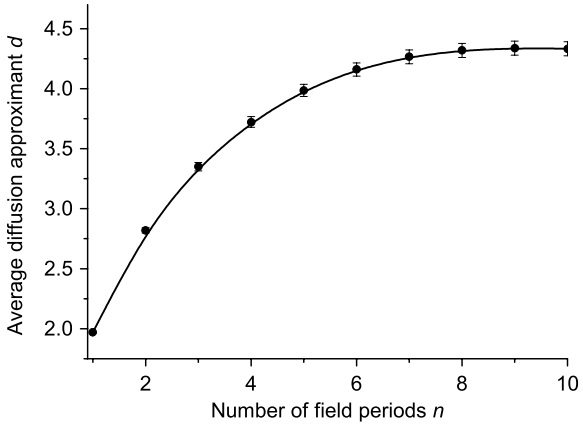

Figure 8. The dependence of the average approximants of the reduced diffusion coefficient d_n on the duration of the time interval measured in periods of the perturbation for the Pullen–Edmonds model (8) on the energy surface $E = 11.0$ for the perturbation frequency $\omega = 1.00$.

increments. One can see that for the values of the time shift larger than $k \approx 5$ the correlations become rather small.

The quantity

$$d_K = \frac{\omega}{4\pi K}\left(\sum_{i=0}^{K-1}\Delta_{n+i}\right)^2 \tag{22}$$

will be called the Kth approximant of the reduced diffusion coefficient. This is a proportionality coefficient between the diffusion coefficient and the square of the field amplitude, calculated from the interval of time of K consequent periods of the field. The positive correlation of Δ_k for small k produces the initial monotonous growth of d_k that eventually comes to a saturation.

From the graph in figure 8, it is seen that already d_8 takes the value that within the 1.5% error margin is undistinguishable from the asymptotic limit. However, this averaged quantity is formed by the contributions that differ by several orders of magnitude. The graph in figure 9 shows the distribution of the quantities d_8 in the log–log scale. The distribution is taken from averaging over four ensembles of 10^5 points each.

The dominating part of this distribution is accurately fitted by the dependence $\ln w = f(d) = -1.771 - 1.057 \ln d - 0.095(\ln d)^2$ that is shown by the thin solid line. For the largest values of d_8 another approximation is valid, $\ln w = g(d) = 19.268 - 6.386 \ln d$. This dependence is shown in figure 9 with the thin dashed line. In the domain of validity of the approximation $f(d)$ the slope of the curve is less than unity: the distribution is of the Zipf–Pareto type, in which the dominating contribution to the average comes from the rare large terms. In our case, 20% of the largest terms come with 82% contribution to the average. These large contributions come from the bits of the trajectories in which the point oscillates almost along the direction of the perturbing force nearly synchronously with the perturbation. Theoretically, the maximal value of d_8 originates from the motion with the law $x(t) = \sqrt{2E}\sin(t)$ and equals to $d_+ = 4\pi E/\omega = 138.23$.

In this resonant case, the energy increment grows linearly in time, that is, ballistically.

Thus, we can indicate a classical counterpart to the quantum dynamics of energy growth. The quantum ballistic bumps are analogous to the nearly resonant bits of the classical trajectories with the quasiballistic energy increase.

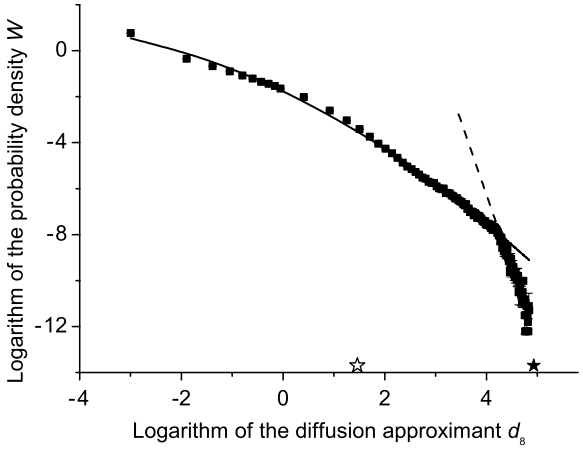

Figure 9. The distribution of the values of partial contributions to the approximant d_8 from different points on the trajectory of the Pullen–Edmonds model (8) on the energy surface $E = 11.0$ for the perturbation frequency $\omega = 1.00$. The thin lines show the approximate forms of the distribution given by the formulae in the text. The black star marks the maximal partial contribution d_+ that comes from the optimal resonant trajectory, while the white star marks the position of the average value $\langle d_8 \rangle$.

4. Conclusion

By the numerical studies of the evolution of the energy distribution in a harmonic external field in a system constructed by the quantization of a classically chaotic Hamiltonian system, thus retaining all correlations of the matrix elements, we have found that the effective rate of the energy diffusion preserves its quadratical dependence on the field strength on the domain of the strong field, where the transition rate is comparable to the perturbation frequency. In other words, the Fermi golden rule appears to be valid far beyond the limits of the domain in which its applicability can be justified. This circumstance restores the quantum–classical correspondence for the energy diffusion and the energy absorption rate in the limit $\hbar \to 0$.

We have to admit that our studies are limited to a specific model, studied only for two values of the perturbation frequency. However, if the mentioned correspondence has to be a universal property, then either the revealed mechanism of the ballistic component of the energy distribution that propagates through the chains of the matrix elements with the correlated signs should be admitted as a general, or some other device should complement it or replace it. This problem deserves further studies.

Acknowledgment

The authors acknowledge the support by the 'Russian Scientific Schools' program (grant no NSh-4464.2006.2).

References

[1] Brenner N and Fishman Sh 1996 *Phys. Rev. Lett.* **77** 3763
[2] Elyutin P V and Shan J 1996 *Phys. Rev. Lett.* **77** 5043
[3] Mukamel Sh, Khidekel V and Chernyak V 1996 *Phys. Rev.* E **53** R1

[4] Cohen D 1999 *Phys. Rev. Lett.* **82** 4951
[5] Cohen D and Kottos T 2000 *Phys. Rev. Lett.* **85** 4839
[6] Iomin A 2000 *Phys. Rev.* E **62** 442
[7] Morawetz K 2000 *Phys. Rev.* E **61** 2555
[8] Reick Ch H 2002 *Phys. Rev.* E **66** 036103
[9] Elyutin P V 2004 *Phys. Rev.* E **69** 036207
[10] Eckhardt B 1988 *Phys. Rep.* **163** 205
[11] Elyutin P V 1988 *Usp. Fiz. Nauk* **155** 397
Elyutin P V 1988 *Sov. Phys. Usp.* **31** 597
[12] Linkwitz S and Grabert H 1991 *Phys. Rev.* B **44** 11888
[13] Jarzynski C 1993 *Phys. Rev.* E **48** 4340
[14] Feingold M and Peres A 1986 *Phys. Rev.* A **34** 591
[15] Wilkinson M 1987 *J. Phys. A: Math. Gen.* **20** 2415
[16] Weisskopf V and Wigner E 1930 *Z. Phys.* **63** 54
[17] Allen L and Eberly J H 1975 *Optical Resonance and Two-Level Atoms* (New York: Wiley)
[18] Delone N B and Krainov V P 1995 *Usp. Fiz. Nauk* **165** 1295
Delone N B and Krainov V P 1995 *Phys. Usp.* **38** 1247
[19] Elyutin P V 2006 *JETP* **102** 182
Elyutin P V 2006 *ZhETF* **129** 207
[20] Basko D M, Skvortsov M A and Kravtsov V E 2003 *Phys. Rev. Lett.* **90** 096801
[21] Kottos T and Cohen D 2001 *Phys. Rev.* E **64** 065202
[22] Komirenko S M, Kim K W, Stroscio M A and Dutta M 2001 *J. Phys.: Condens. Matter* **13** 6233
[23] Pullen R A and Edmonds A R 1981 *J. Phys. A: Math. Gen.* **14** L477
[24] Meyer H-D 1986 *J. Chem. Phys.* **84** 3147
[25] Vorobyev P A and Zaslavsky G M 1987 *Sov. Phys.—JETP* **65** 877
Vorobyev P A and Zaslavsky G M 1987 *ZhETF* **92** 1564
[26] Elyutin P V and Korolev V G 1989 *Moscow Univ. Phys. Bull.* **44** 106
Elyutin P V and Korolev V G 1989 *Vestn. Mosk. Univ. 3* **30** 87
[27] Shnirelman A I 1974 *Usp. Mat. Nauk* **29** 181